We develop a mathematical theory for finance based on the “viability” principle that it should not be possible to fund, starting with arbitrarily small initial capital, a nonnegative liability which is strictly positive with positive probability. In the context of continuous semimartingale asset prices, we show that proscribing such egregious arbitrages (but allowing for the possibility that one portfolio might outperform another) turns out to be equivalent to each of the following conditions:

(i) a portfolio with the local martingale numeraire property exists, (ii) a growth-optimal portfolio exists, (iii) a portfolio with the log-optimality property exists, (iv) a strictly positive local martingale deflator exists, (v) the market has locally finite maximal growth.

These equivalent conditions can be formulated in terms of the characteristics of the underlying asset prices. Theories for hedging and optimization, and the important notion of “market completeness”, are developed in such a setting. The semimartingale property of asset prices is equivalent to viability, when investment is discrete and long-only. (Joint work-book of the same title—with Constantinos Kardaras.) (Received September 12, 2018)