Fluid–solid interaction problems are encountered in many engineering and biological applications, but are challenging to simulate due to the coupling between the two material phases. Typically, solids are simulated using a Lagrangian approach with a grid that moves with the material, whereas fluids are simulated using an Eulerian approach with a fixed spatial grid, requiring some type of interfacial coupling between the two different perspectives. Here, we present a fully Eulerian method for simulating structures immersed in a fluid. By introducing a reference map variable to model finite-deformation constitutive relations in the structures on the same grid as the fluid, the interfacial coupling problem is highly simplified. The method is particularly well suited for simulating soft, highly-deformable materials and many-body contact problems. We also extend the technique to simulate rigid solids in an incompressible fluid, using a projection step formulated as a composite linear system that simultaneously enforces the rigidity and incompressibility constraints. Several examples including single deformable/rigid objects, multiple objects and soft-rigid combinations will be presented. (Received September 24, 2018)