Franklin Kenter and Daphne Skipper* (skipper@usna.edu). *IP bounds on pebbling numbers of Cartesian-product graphs.

A pebbling move takes two pebbles from a single vertex in a graph and places one pebble on an adjacent vertex. The pebbling number of a graph $G$ is the smallest number $\pi_G$ such that, given any vertex $k$ of $G$ and any assignment of $\pi_G$ pebbles to the vertices of $G$, there exists a sequence of pebbling moves that places a pebble on $k$. Computing $\pi_G$ is provably difficult. Graham’s conjecture states that the pebbling number of the Cartesian-product of two graphs $G$ and $H$, denoted $G \Box H$, is no greater than $\pi_G \pi_H$.

This study combines the focus of developing a computationally tractable method for generating good bounds on $\pi_G \Box H$, with the goal of providing evidence for (or disproving) Graham’s conjecture. In particular, we present a novel integer-programming (IP) approach to bounding $\pi_G \Box H$ that results in significantly smaller problem instances compared with existing IP approaches to graph pebbling. Our approach leads to an improvement on the best known bound for $\pi_L \Box L$, where $L$ is the Lemke graph. $L \Box L$ is among the smallest known potential counterexamples to Graham’s conjecture. (Received September 17, 2018)