We present a first order local regularization method for solving ill-posed Volterra equations with $\nu$-smoothing kernels. Based upon a previous method that performs well only for one-, two-, and three-smoothing kernels, our method is shown to be convergent for all values of $\nu \in \mathbb{N}$. We describe numerical implementation of the method and provide a new scheme to approximate the initial condition. Numerical examples illustrate the newly achieved stability in the cases $\nu = 4$ and higher. (Received September 25, 2018)