Large arboreal Galois representations.

Given a field $K$, a polynomial $f \in K[x]$ of degree $d$, and a suitable element $t \in K$, the set of preimages of $t$ under the iterates $f^\circ n$ carries a natural structure of a complete rooted $d$-ary tree $T_\infty$. The Galois action on the roots of $f^\circ n(x) - t$ gives rise to a homomorphism $\phi : G_K \to \text{Aut}(T_\infty)$ known as the arboreal Galois representation attached to $f$ and $t$. Arboreal representation is an arithmetic dynamics analogue of the Tate module. We study conditions under which the representation $\phi$ is surjective. For $d$ even we prove a criterion relating the surjectivity of $\phi$ with the arithmetic of the critical orbit of $f$. When $d \geq 20$ is even we use this criterion to exhibit examples of polynomials with maximal Galois action on the preimage tree, partially affirming a conjecture of Odoni (simultaneously and independently of our work two papers on Odoni’s conjecture appeared; the full conjecture was proved by Joel Specter; Robert Benedetto and Jamie Juul proved the conjecture for most number fields). We also study the case of $K = F(t)$ and $f \in F[x]$ in which the corresponding Galois groups are the monodromy groups of ramified covers $f^\circ n : \mathbb{P}^1_F \to \mathbb{P}^1_F$. (Received September 02, 2018)