In a paper published in 1856, G.G. Stokes (1819-1903) used a divergent series to compute many values of the Airy integral. Some of these values had been previously computed via a convergent series but this method was too laborious to make all of the desired calculations. This talk will examine how Stokes numerically computed a class of definite integrals, including the Airy integral, using divergent infinite series. Emphasis will be placed on what lead Stokes to use this method, what types of physical problems required these solutions, how Stokes justified using his method, and how the results obtained were verified. How, when and for what purpose did other mathematicians and physicists use this method during the mid-19th century, before divergent series were given a rigorous treatment, will also be discussed. (Received September 15, 2019)