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Wesley H. Holliday* (wesholliday@berkeley.edu). *Axiomatizing reasoning about sets: cardinality, mereology, and decisiveness.*

In this talk, I give three examples of axiomatizing reasoning about sets in special purpose languages. First, I consider reasoning about comparative cardinality: $A \geq B$ if there is an injection from B to A . I add principles to Boolean algebra to axiomatize reasoning not only about Boolean operations but also about \geq . Second, I consider reasoning about the subset relation (“set-theoretic mereology”) in a modal language: $\Diamond\varphi$ is true at a set A if there is a nonempty $B \subseteq A$ such that φ is true at B . I discuss the longstanding open problem of giving a recursive axiomatization of the set of validities for finite sets. Finally, I give an example outside of pure mathematics from voting theory: a set A of voters is decisive over candidates x, y if whenever all voters in A prefer x to y , society must rank x above y . I present an axiomatization of reasoning about decisive sets of voters for voting methods satisfying well-known axioms. These examples are meant to illustrate a methodology familiar to modal logicians: to better understand the core principles governing some mathematical concept, try to axiomatize the validities of a lean language with dedicated operators whose semantics is given by the target concepts. (Received September 16, 2019)