Mozhgan Mirzaei* (momirzae@ucsd.edu) and Andrew Suk. Extremal Configurations in Point-Line Arrangements. Preliminary report.

The famous Szemerédi-Trotter theorem states that any arrangement of \( n \) points and \( n \) lines in the plane determines \( O(n^{4/3}) \) incidences, and this bound is tight. In this talk, we present some Turán-type results for point-line incidences.

Let \( L_1 \) and \( L_2 \) be two sets of \( t \) lines in the plane and let \( P = \{ \ell_1 \cap \ell_2 : \ell_1 \in L_1, \ell_2 \in L_2 \} \) be the set of intersection points between \( L_1 \) and \( L_2 \). We say that \((P, L_1 \cup L_2)\) forms a natural \( t \times t \) grid if \(|P| = t^2\), and \( \text{conv}(P) \) does not contain the intersection point of some two lines in \( L_i \), for \( i = 1, 2 \). For fixed \( t > 1 \), we show that any arrangement of \( n \) points and \( n \) lines in the plane that does not contain a natural \( t \times t \) grid determines \( O(n^{4/3-\varepsilon}) \) incidences, where \( \varepsilon = \varepsilon(t) \). We also provide a construction of \( n \) points and \( n \) lines in the plane that does not contain a natural \( 2 \times 2 \) grid and determines at least \( \Omega(n^{1+\frac{1}{14}}) \) incidences. This is joint work with Andrew Suk. (Received August 13, 2019)