Let \( q \) be a prime power and \( \mathbb{F}_{q} \) be the field of \( q \) elements. For \( i = 2, 3 \), let \( P_i = L_i = \mathbb{F}_{q}^i \) and \( f_2 \in \mathbb{F}_{q}[X_1, Y_1], f_3 \in \mathbb{F}_{q}[X_1, Y_1, X_2, Y_2] \) be polynomials over \( \mathbb{F}_{q} \). We define \( \Gamma_2 = \Gamma(q; f_2) \) to be the bipartite graph with partition sets \( P_2 \) and \( L_2 \) such that \( p = (p_1, p_2) \in P_2 \) and \( l = [l_1, l_2] \in L_2 \) are adjacent if and only if
\[
p_2 + l_2 = f_2(p_1, l_1).
\]
Similarly, we define \( \Gamma_3 = \Gamma(q; f_2, f_3) \) to be the bipartite graph with partition sets \( P_3 \) and \( L_3 \) such that \( p = (p_1, p_2, p_3) \in P_3 \) and \( l = [l_1, l_2, l_3] \in L_3 \) are adjacent if and only if
\[
p_2 + l_2 = f_2(p_1, l_1)
\]
\[
p_3 + l_3 = f_3(p_1, l_1, p_2, l_2).
\]
The canonical projection \( \Phi : \mathbb{F}_{q}^{3} \to \mathbb{F}_{q}^{2}, \langle v_1, v_2, v_3 \rangle \leftrightarrow \langle v_1, v_2 \rangle \) induces a surjective \( q \)-to-1 map \( \Phi : V(\Gamma_3) \to V(\Gamma_2) \) by \( (p_1, p_2, p_3) \mapsto (p_1, p_2) \) and \( [l_1, l_2, l_3] \mapsto [l_1, l_2] \). This map \( \Phi \) is a graph homomorphism, and so we call \( \Gamma_3 \) a \( q \)-lift of \( \Gamma_2 \).

We present some properties of \( q \)-lifts and explain how their specializations relate to finite geometries. (Received September 15, 2019)