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**Colin Defant\*** (cdefant@princeton.edu), **Michael Engen** and **Jordan A Miller**. *Uniquely Sorted Permutations and Fertility Numbers*.

The *fertility* of a permutation  $\kappa$  is  $|s^{-1}(\kappa)|$ , where  $s$  denotes West's stack-sorting map. A permutation is *uniquely sorted* if its fertility is 1. It turns out that there are no uniquely sorted permutations of even length. We show that uniquely sorted permutations of odd length are counted by Lassalle's sequence; this mysterious sequence emerged when Lassalle used algebraic methods to prove that its terms are positive and increasing, settling a conjecture of Zeilberger. Our enumeration of uniquely sorted permutations follows from a more general bijection that links new combinatorial objects called "valid hook configurations" with weighted set partitions and cumulants appearing in free probability theory. We also briefly discuss some bijective enumerations of pattern-avoiding valid hook configurations and uniquely sorted permutations, including recent (separate) works of Mularczyk and Sankar. We then turn our attention to the set of nonnegative integers that arise as fertilities of permutations, revealing some unexpected patterns and several open problems. (Received August 17, 2019)