We introduce a family of ideals $I_{n,\lambda}$ in $\mathbb{Q}[x_1, \ldots, x_n]$ for $\lambda$ a partition of $k \leq n$. This family contains both the Tanisaki ideals $I_\lambda$ and the ideals $I_{n,k}$ of Haglund-Rhoades-Shimozono as special cases. We study the corresponding quotient rings $R_{n,\lambda}$ as symmetric group modules. When $n = k$, we recover the Garsia-Procesi modules, and when $\lambda = (1^k)$, we recover the generalized coinvariant algebras of Haglund-Rhoades-Shimozono.

We will present a monomial basis for $R_{n,\lambda}$ in terms of $(n, \lambda)$-staircases, unifying the monomial bases studied by Garsia-Procesi and Haglund-Rhoades-Shimozono. Furthermore, we realize the $S_n$-module structure of $R_{n,\lambda}$ in terms of an action on $(n, \lambda)$-ordered set partitions. We will then show that the graded Frobenius characteristic of $R_{n,\lambda}$ has a positive expansion in terms of dual Hall-Littlewood functions. We also show that the rings $R_{n,\lambda}$ have connections to the rank varieties of Eisenbud-Saltman. We then generalize results of De Concini-Procesi and Tanisaki on “nilpotent” diagonal matrices. (Received September 17, 2019)