Large Rank Numbers & $(K_s - e) \times P_n$.

A $k$-ranking of a graph $G$ is a function $f : V(G) \to \{1, 2, \ldots, k\}$ such that if $f(u) = f(v)$ then every $uv$ path contains a vertex $w$ such that $f(w) > f(u)$. The rank number of $G$, denoted $\chi_r(G)$, is the minimum $k$ such that a $k$-ranking exists for $G$. The rank number is a variant of graph colorings. It is known that given a graph $G$ and a positive integer $t$ the question of whether $\chi_r(G) \leq t$ is NP-complete. The characteristics of any $n$-vertex graph whose rank number is equal to $n - 1$ or $n - 2$ is known; in this talk we extend this question to $n - 3$. Also, we examine the extremal graphs such that their rank number is equal to $n$, $n - 1$, $n - 2$ and $n - 3$.

The ranking of $K_s \times P_n$ has been previously studied, and a recursive formula for $\chi_r(K_s \times P_n)$ has been established. In this talk, we study the ranking of $K_s - e \times P_n$. We establish the rank number of $K_s - e \times P_n$ for even $s \geq 4$ and provide a conjecture for $\chi_r(K_n - e \times P_n)$ for odd $s \geq 5$. (Received July 26, 2019)