We will introduce “stability” versions of upper bounds on maximal independent set counts in graphs under various restrictions. Roughly these say that being close to the maximum implies existence of a large induced matching or triangle matching (depending on assumptions). We will also discuss applications and related open problems in various directions. (Received September 11, 2019)