

1154-11-1093      **Max Alekseyev\*** (maxal@gwu.edu) and **Neil Sloane** (njasloane@gmail.com). *Combinatorics of Kaprekar's Junction Numbers*. Preliminary report.

A base  $b$  junction number  $u$  has the property that there are at least two ways to write it as  $u = v + s(v)$ , where  $s(v)$  is the sum of the digits in the expansion of the number  $v$  in base  $b$ . For the base 10 case, Kaprekar in the 1950's and 1960's studied the problem of finding  $K(n)$ , the smallest  $u$  such that the equation  $u = v + s(v)$  has exactly  $n$  solutions. He gave the values  $K(2) = 101$ ,  $K(3) = 10^{13} + 1$ , and conjectured that  $K(4) = 10^{24} + 102$ . In 1966 Narasinga Rao gave the upper bound  $10^{11111111111124} + 102$  for  $K(5)$ , as well as upper bounds for  $K(6)$ ,  $K(7)$ ,  $K(8)$ , and  $K(16)$ . We will present a set of recurrences which determine  $K(n)$  for any base  $b$ , and in particular we will show that these conjectured values of  $K(n)$  are correct. Rather surprisingly, the solution to the base 2 problem is determined by the classical Thue-Morse sequence. For fixed  $b$ , it appears that  $K(n)$  grows as a tower of height about  $\log_2(n)$ . (Received September 13, 2019)