A base $b$ junction number $u$ has the property that there are at least two ways to write it as $u = v + s(v)$, where $s(v)$ is the sum of the digits in the expansion of the number $v$ in base $b$. For the base 10 case, Kaprekar in the 1950’s and 1960’s studied the problem of finding $K(n)$, the smallest $u$ such that the equation $u = v + s(v)$ has exactly $n$ solutions. He gave the values $K(2) = 101$, $K(3) = 10^{13} + 1$, and conjectured that $K(4) = 10^{24} + 102$. In 1966 Narasinga Rao gave the upper bound $10^{111111111124} + 102$ for $K(5)$, as well as upper bounds for $K(6)$, $K(7)$, $K(8)$, and $K(16)$. We will present a set of recurrences which determine $K(n)$ for any base $b$, and in particular we will show that these conjectured values of $K(n)$ are correct. Rather surprisingly, the solution to the base 2 problem is determined by the classical Thue-Morse sequence. For fixed $b$, it appears that $K(n)$ grows as a tower of height about $\log_2(n)$. (Received September 13, 2019)