Ben Kane and Jingbo Liu* (jliu@tamusa.edu). Universal sums of generalized $m$-gonal numbers.

Conway–Schneeberger Fifteen Theorem states that a given positive definite integral quadratic form is universal (i.e., represents every positive integer) if and only if it represents all the positive integers up to 15. We are interested in generalizing this question to sums of generalized $m$-gonal numbers with positive coefficients:

$$f(x) = \sum_{j=1}^{n} a_j P_m(x_j)$$

where

$$P_m(x) := \frac{(m - 2)x^2 - (m - 4)x}{2}, \quad x \in \mathbb{Z}.$$ 

Let $\gamma(m)$ be the smallest positive integer such that $f$ is universal if and only if every positive integer less than or equal to $\gamma(m)$ is represented by $f$. We have known that $\gamma(3) = \gamma(6) = 8$ and $\gamma(4) = 15$. Recently Ju and Oh have proven that $\gamma(8) = 60$. In this talk, we will approach this problem from both algebraic and analytic sides and determine an asymptotic upper bound, as a function of $m$, for $\gamma(m)$.

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