In 1922, Mordell conjectured the striking statement that for a polynomial equation $f(x, y) = 0$, if the topology of the set of complex number solutions is complicated enough, then the set of rational number solutions is finite. This was proved by Faltings in 1983, and again by a different method by Vojta in 1991, but neither proof provided a way to provably find all the rational solutions, so the search for other proofs has continued. Recently, Lawrence and Venkatesh found a third proof, relying on variation in families of $p$-adic Galois representations; this is the subject of the lecture. (Received September 15, 2019)