On the inductive construction of rank 1, 2, and 3 valuations on $K(x, y)$ and their associated structures.

In a 1935 paper, Saunders Maclane constructed all rank one and some rank two valuations on the field of rational functions $K(x)$ extending a discrete valuation $v_0$ on a field $K$ via inductively defined sequences of key polynomials in $K[x]$, and in 2009, K.A. Loper and F. Tartarone introduced the notion of an "upside down" valuation to construct the remaining rank two valuations on $K(x)$ extending $v_0$ on $K$ and used both Maclane’s construction and their own to represent integrally closed rings between $\mathbb{Z}_p[x]$ and $\mathbb{Q}[x]$ as a minimal intersection of valuation overrings and classify such rings (when they are Noetherian, Prüfer, Mori, PvMD, etc.) by the form of the valuations corresponding to the valuation overrings in that minimal intersection. In a series of 3 papers published in 1969, 1971, and 1973, H. Inoue attempted to generalize Maclane’s approach to $K(x, y)$ but was not able to fully accomplish this goal. This talk will present fixes to the problems in Inoue’s approach and some results on the construction of rank two and rank 3 valuations on $K(x, y)$ and integrally closed rings between $\mathbb{Z}_p[x, y]$ and $\mathbb{Q}[x, y]$. (Received September 16, 2019)