Grigorchuk and Zuk (2000) realized the lamplighter group $\mathbb{Z}_2 \wr \mathbb{Z}$ as an automaton group and performed spectral computations that led to a counterexample to the strong Atiyah conjecture on $\ell_2$-betti numbers.

Shortly after Silva and Steinberg gave a construction of $A \wr \mathbb{Z}$ as an automaton group for any finite abelian group $A$ using reversible automata and a representation as rational power series over finite rings.

In 2015 Bondarenko, D’Angeli and Rodaro realized the lamplighter group $\mathbb{Z}_3 \wr \mathbb{Z}$ as a bireversible automaton group. Previously known bireversible automaton groups tended to be non-amenable or virtually nilpotent, so this was quite a surprise.

Bondarenko and Savchuk announced a construction of $A \wr \mathbb{Z}$ as a bireversible automaton group for $A$ an elementary abelian $p$-group with $p$ an odd prime using rational power series over finite fields.

In this talk we discuss realizations of lamplighter groups $A \wr \mathbb{Z}$ with $A$ finite abelian as rational series over finite rings. We show that bireversibility seems to impose a constraint on the 2-Sylow subgroups. (Received September 13, 2019)