For each directed graph we construct an associated inverse semigroup that we call the Leavitt inverse semigroup of the graph. Leavitt inverse semigroups are closely related to Leavitt path algebras and graph inverse semigroups. They provide a certain amount of structural information about Leavitt path algebras. For example two graphs that have isomorphic Leavitt inverse semigroups have isomorphic Leavitt path algebras, but the converse is false. We study some topological aspects of the structure of graph inverse semigroups and Leavitt inverse semigroups and we provide necessary and sufficient conditions for two graphs to have isomorphic Leavitt inverse semigroups. (Received September 04, 2019)