It is a profound result of Hölder in 1887 that the Euler gamma-function cannot satisfy any nontrivial algebraic differential equation in $\mathbb{C}$. Hilbert, in his lecture addressed before the International Congress of Mathematicians at Paris in 1900 for his famous 23 problems, stated in Problem 18 that the Riemann zeta-function cannot satisfy any such a differential equation either.

In 2007, Markus showed that $\zeta(\sin(2\pi z))$ cannot satisfy such a differential equation with polynomial coefficients in $\Gamma$ and its derivatives; he conjectured $\zeta$ itself cannot satisfy such a differential equation with polynomial coefficients in $\Gamma$ and its derivatives either.

Thus, one wonders if there is a nontrivial polynomial $P(u_0, u_1, \ldots, u_m; v_0, v_1, \ldots, v_n)$ such that

$$P(\zeta, \zeta', \ldots, \zeta^{(m)}; \Gamma, \Gamma', \ldots, \Gamma^{(n)}) \equiv 0. \quad (1)$$

In this joint work with Dr. Jingbo Liu, we show that $\zeta$ and $\Gamma$ cannot satisfy some differential equations generated through a family $\mathcal{F}$ of functions $\mathcal{F}(u_0, u_1, \ldots, u_m; v_0, v_1, \ldots, v_n)$ which are continuous in $(u_0, u_1, \ldots, u_m)$ with polynomial coefficients of $(v_0, v_1, \ldots, v_n)$. (Received September 11, 2019)