The Navier-Stokes and the Euler equations are the fundamental models describing incompressible homogenous fluids, with and without viscosity. Based on theories of fluid turbulence, one may expect that solutions of these equations have on average a finite degree of smoothness, in the infinite Reynolds number limit. These ideas go back to Kolmogorov ’41 and to Onsager ’49. As such, it is natural to consider weak or distributional solutions of the fluid equations. The behavior of weak solutions to the Navier-Stokes and Euler equations is however mysterious: the equations appear to be too soft at this low smoothness level. One may exhibit weak solutions which have finite kinetic energy, and have compact support in time: the fluid is fully at rest, then it starts to move, and then it goes back to sleep. In this talk, we survey a number of results concerning such wild weak solutions of the fluid equations. These works build on the groundbreaking works of De Lellis and Szekelyhidi Jr., who extended Nash’s fundamental ideas on $C^1$ flexible isometric embeddings, into the realm of fluid dynamics. These techniques, which go under the umbrella name ”convex integration”, have fundamental analogies the phenomenological theories of turbulence. (Received September 01, 2019)