Even though the interaction between fluids and solids is one of the most classical problems in fluid mechanics, mathematicians have only recently begun developing a systematic theory to study this class of problems. The strong nonlinearities make the coupled, nonlinear moving boundary problem(s) exceedingly difficult to study. In this talk I will survey the most recent developments in this area, and show how the mathematical theory we are developing influences real-life problems, such as optimal design of cardiovascular prostheses, and optimal design of bioartificial pancreas. (Received September 17, 2019)