

1154-35-279

**Michael Benfield, Helge Kristian Jenssen and Irina A Kogan\*** (iakogan@ncsu.edu). *A Generalization of an Integrability Theorem of Darboux.*

In his book “Systèmes Orthogonaux” (1910), Darboux stated a theorem (Théorème III) on local existence and uniqueness of solutions to PDE systems of the type

$$\partial_{x_i} u_\alpha(x) = f_i^\alpha(x, u(x)), \quad i \in I_\alpha \subseteq \{1, \dots, n\}, \quad \alpha = 1, \dots, m,$$

where, for a given point  $\bar{x} \in \mathbb{R}^n$ , the values for the unknown  $u_\alpha$  are prescribed near  $\bar{x}$  along  $\{x \mid x_i = \bar{x}_i \text{ for each } i \in I_\alpha\}$ . The theorem was proven by Darboux only for  $n = 2$  and  $3$ . We prove a generalization of Darboux’s result, applicable to

$$\mathbf{r}_i(u_\alpha)|_x = f_i^\alpha(x, u(x)), \quad i \in I_\alpha \subseteq \{1, \dots, n\}, \quad \alpha = 1, \dots, m,$$

where  $\{\mathbf{r}_i\}_{i=1}^n$  is a local frame of vector fields. The values for  $u_\alpha$  are prescribed along a manifold  $\Xi_\alpha$  transverse to the vector fields  $\{\mathbf{r}_i \mid i \in I_\alpha\}$ . We identify a geometric condition, the Stable Configuration Condition (SCC), that depends on both the frame and the data manifolds. Assuming the SCC and the integrability conditions are satisfied, we establish local existence and uniqueness of a  $C^1$ -solution for arbitrary  $n$ . If the SCC is not satisfied, we show that the uniqueness may fail. (Received August 28, 2019)