Given a dynamical portrait for several rational functions acting on a finite set, we initiate the study of the corresponding moduli space of realizations. In this talk we discuss the various ways in which these moduli spaces can have higher than expected dimension and present results explaining this phenomenon. Time permitting we will also survey the findings of large exhaustive computations we conducted for portrait spaces of cubic polynomials. (Received September 12, 2019)