In the study of translation surfaces, a fundamental result is that any geodesic is either closed (homeomorphic to $S^1$), a saddle connection (homeomorphic to an interval), or dense in some subsurface (possibly with boundary). Homothety surfaces (also called dilation surfaces) are a mild generalization of translation surfaces, whose dynamical properties are just beginning to be explored. I will describe a family of genus-2 homothety surfaces on which every geodesic either accumulates on a closed loop or is dense in a lamination with Cantor cross-section. The Cantor sets that arise all have Hausdorff dimension zero, but their geometric structures are determined by continued fractions. This is based on joint work with Slade Sanderson. (Received September 17, 2019)