To obtain matrix-valued Aleksandrov–Clark (AC) measures, fix a matrix-valued pure contraction $b$ on $\mathbb{D}$. Here $b$ may be non-inner and/or non-extreme. For each unitary matrix $\alpha$, the linear fractional transformation $(I + b(z)\alpha^*)(I - b(z)\alpha^*)^{-1}$ has non-negative real part. So, Herglotz’s representation theorem associates a matrix-valued measure $\mu^\alpha$. The collection $\{\mu^\alpha : \alpha \text{ unitary}\}$ forms the family of matrix-valued AC measures, which stand in bijection with matrix-valued pure contractions. A description of the measures’ absolutely continuous parts is easily obtained in terms of non-tangential boundary values of $b$.

The singular parts $\mu^\alpha_s$ are harder. We present a matrix-valued version of Nevanlinna’s result relating non-tangential boundary limits with the measures’ point masses. The connection to Carathéodory angular derivatives is more subtle than in the scalar setting. Aleksandrov spectral averaging yields restrictions on the singular parts. We have a directional version of the mutual singularity of $\mu^\alpha_s$ and $\mu^\beta_s$, $\alpha \neq \beta$ both unitary. This presentation is based on joint work with R.T.W. Martin and S. Treil. (Received August 28, 2019)