Doubly transitive lines: Symmetry implies optimality.

Since the work of L.F. Tóth on regular figures, it has been widely observed that optimal solutions to packing problems frequently display extraordinary symmetries. For instance, spheres centered on points in the Leech lattice give an optimal packing in 24 dimensions, while lines through antipodal vertices of an icosahedron give an optimal packing in two-dimensional projective space. In this talk, we demonstrate an extreme case of this phenomenon for line packings: symmetry can be a sufficient condition for optimality. Specifically, consider $n$ lines spanning a space of dimension $d < n$. If the lines have a doubly transitive automorphism group, then they are optimally packed in projective space. In fact, unit norm representatives for the lines reach equality in the Welch bound to create an equiangular tight frame. We will explain this phenomenon, and then discuss progress toward a classification of all doubly transitive lines. (Received September 09, 2019)