For $G_2$-structures on 7-manifolds, there is a natural analog of the Ricci-flow studied in Riemannian geometry, namely, one considers a 1-parameter family $\sigma = \sigma(t)$ of $G_2$-structures on a given 7-manifold that satisfies the equation

$$\frac{d\sigma}{dt} = \Delta_\sigma \sigma$$

with a specified initial $G_2$-structure $\sigma(0) = \sigma_0$.

When the 1-parameter family $\sigma$ moves by diffeomorphism and scaling, we say that $\sigma$ is a soliton for the $G_2$-Laplacian flow. The most interesting case is when the initial $G_2$-structure is closed.

In this talk, I will describe some of what is known about the existence and local generality of solitons for this flow, concluding with a discussion of the still-unsolved problem of the generality of the gradient solitons, which are of great interest in the theory of $G_2$-structures. (Received August 26, 2019)