Topological data analysis studies the topology of spaces obtained by thickening a set of data. For example, if the data is a metric space $M$ then one can build the Vietoris–Rips simplicial complex on $M$. This can be viewed as a subset of the space $\mathcal{P}(M)$ of probability measures on $M$. We study the topology of $\mathcal{P}(M)$. In particular, if $M$ is a manifold and $\mathcal{P}(M)$ is equipped with the 2-Wasserstein distance, it inherits a type of differential structure from $M$. Using ideas inspired by classical Morse theory, we determine the homotopy type of certain Wasserstein spaces. (Received September 17, 2019)