Computations in $RO(G)$-graded Bredon cohomology can be challenging and are not well understood, even for $G = C_2$, the cyclic group of order two. A recent structure theorem for $RO(C_2)$-graded cohomology with coefficients in the constant Mackey functor $\mathbb{F}_2$ substantially simplifies computations. The structure theorem says the cohomology of any finite $C_2$-CW complex decomposes as a direct sum of two basic pieces: cohomologies of representation spheres and cohomologies of spheres with the antipodal action. This decomposition lifts to a splitting at the spectrum level. In joint work with Dan Dugger and Christy Hazel we extend this result to a classification of compact modules over the Eilenberg-MacLane spectrum $H\mathbb{F}_2$. (Received September 14, 2019)