Towards Directed Collapsibility.

In the directed setting, the spaces of directed paths between fixed initial and terminal points are the defining feature for distinguishing different directed spaces. The simplest case is when the space of directed paths is homotopy equivalent to that of a single path; we call this the trivial space of directed paths. Directed spaces that are topologically trivial may have non-trivial spaces of directed paths, which means that information is lost when the direction of these topological spaces is ignored. We define a notion of directed collapsibility in the setting of a directed Euclidean cubical complex using the spaces of directed paths of the underlying directed topological space relative to an initial or a final vertex. In addition, we give sufficient conditions for a directed Euclidean cubical complex to have a contractible or a connected space of directed paths from a fixed initial vertex. We also give sufficient conditions for the path space between two vertices in a Euclidean cubical complex to be disconnected. Our results have applications to speeding up the verification process of concurrent programming and to understanding partial executions in concurrent programs. (Received September 16, 2019)