A domination in a category $\mathcal{C}$ is a morphism $f : X \rightarrow Y$, $X, Y \in \text{Ob}\mathcal{C}$, such that there exists a morphism $g : Y \rightarrow X$ with $fg = \text{id}_Y$. Then $Y$ is dominated by $X$ (we write $X \geq Y$). In the following, $\mathcal{C}$ is the homotopy category of $CW$-complexes, or the shape category of compacta.

Let $P$ be a polyhedron. We may ask, if each sequence $P \geq X_1 \geq X_2 \geq \ldots$ contains only finitely many different homotopy types of $X_i$ or, does there exist an integer $l_P$ (depending only on $P$) such that each sequence as above contains $\leq l_P$ different homotopy types? The answers are known only in dimension 1. It should be noted that there exist polyhedra (even 2-dimensional) dominating infinitely many different homotopy types or shapes (DK, 1996).

These questions are closely related to the problems of K. Borsuk (1967, 1975): “Is it true that two $ANR$'s homotopy dominating each other have the same homotopy type?” and ”Are the homotopy types of two quasi-homeomorphic $ANR$'s equal?”

We will show that for some polyhedra (including all 2-dimensional polyhedra) the answers depend only on the properties of the fundamental group $\pi_1(P)$, and will present new positive results. (Received September 17, 2019)