A multiplicatively symmetrized version of the Chung-Diaconis-Graham random process. Preliminary report.

This work looks at random processes of the form $X_{n+1} = a_nX_n + b_n \pmod{p}$ where $(a_0, b_0), (a_1, b_1), (a_2, b_2), ...$ are i.i.d. with $P(a_n = (p+1)/2) = P(a_n = 2) = 1/2$ and $P(b_n = 1) = P(b_n = 0) = P(b_n = -1) = 1/3$, $p$ is odd, and $X_0 = 0$. This work shows that order $(\log p)^2$ steps are sufficient for $X_n$ to be close to uniformly distributed on the integers mod $p$. Also order $(\log p)^2$ steps are necessary for $X_n$ to be close to uniformly distributed in the integers mod $p$. A consequence is that there are doubly stochastic matrices $P_1$ and $P_2$ such that at least one row of $(0.5P_1 + 0.5P_2)^m$ will be close to uniform only for $m$ much larger than values of $m$, i.e. order $(\log p)\log(\log p)$, which suffice to make all rows of $P_1^m$ and $P_2^m$ close to uniform. (Received September 12, 2019)