A (almost-) square permutation is a permutation where (almost) all points are records, either maximums or minimums, from the left or the right. We give a probabilistic approach to computing the first-order enumeration of almost-square permutations of size $n + k$ with $n$ records and $k$ non-records. We use the language of permuton limits to describe the points of an almost-square permutation scaled to fit in the unit square. A uniformly random almost-square permutation has a permuton limit with a simple geometric description. If $k$ is fixed, the permuton can be viewed as a rectangle embedded in $[0,1]^2$ whose edges have slope $\pm 1$ and whose bottom corner is given by a $\beta(k+1, k+1)$ random variable. The bottom corner is uniform if $k = 0$ and concentrated at $1/2$ if $k$ is large. If $k$ is increasing but small relative to $n$, then the permuton limit is a square. (Received September 11, 2019)