Fibre bundles serve as a natural geometric setting for many learning problems involving non-scalar pairwise interactions. Modeled on a fixed principal bundle, different irreducible representations of the structural group induce many associated vector bundles, encoding rich geometric information for the fibre bundle as well as the underlying base manifold. An intuitive example for such a learning paradigm is phase synchronization—the problem of recovering angles from noisy pairwise relative phase measurements—which is prototypical for a large class of imaging problems in cryogenic electron microscopy (cryo-EM) image analysis. We propose a novel nonconvex optimization formulation for this problem, and develop a simple yet efficient two-stage algorithm that, for the first time, guarantees strong recovery for the phases with high probability. We demonstrate applications of this multi-representation methodology that improve denoising and clustering results for cryo-EM images. This algorithmic framework also extends naturally to general synchronization problems over other compact Lie groups, with a wide spectrum of potential applications. (Received August 16, 2019)