A nonlocal diffuse interface model with a nonsmooth potential.

We propose a nonlocal variant of the Cahn–Hilliard diffuse interface model with a nonsmooth potential of double-well obstacle type. We discuss different variants of the nonlocal contributions in the model such as nonlocal operators and nonlocal boundary conditions. In contrast to the local setting, the proposed model allows for sharp interfaces in the solution for a certain critical (non-zero) value of the interface parameter. Here, the choice of the obstacle potential plays an important role in our analysis, since it guarantees the strict separation of the substance into pure phases for nontrivial interactions. Mathematically, this introduces additional inequality constraints that, in a weak form, lead to a coupled system of variational inequalities, which at each time instance can be restated as a constrained optimization problem.

We analyze a discretization of the problem in space and time based on finite elements and implicit-explicit time stepping methods that can be realized efficiently. We provide numerical experiments to support our theoretical findings in one and two spatial dimensions. (Received September 16, 2019)