Reproductive numbers, metastability and extinction in discrete-time models of endemic diseases.

For epidemiological models, the basic reproductive number $R_0$ corresponds to the expected number of individuals infected by an infected individual in a mostly susceptible population. For deterministic models of endemic diseases, $R_0 > 1$ often implies that the disease will persist indefinitely. In contrast, $R_0 < 1$ often implies that the disease goes extinct asymptotically. In sharp contrast, the stochastic counterparts of these deterministic models predict that the diseases go extinct in finite time whether or not $R_0 > 1$. To understand this discrepancy, we analyzed the quasi-stationary distributions (QSDs) for discrete-time Markov chain models of endemic diseases. Mathematically, QSDs correspond a left eigenvector of the transition matrix associated with dominant eigenvalue $\lambda < 1$. Biologically, QSDs describe the long-term statistical behavior of the disease conditioned on non-extinction and $1/(1 - \lambda)$ is the mean time extinction (MTE) when following the QSD. We show that (i) $R_0 > 1$ implies that the MTE increases exponentially with the population size $N$, while (ii) $R_0 < 1$ implies that the MTE increases and saturates with $N$. We also prove the QSDs are concentrated on the attractors of the corresponding deterministic models when $N$ is large. (Received September 17, 2019)