Euler used intrinsic equations expressing the radius of curvature as a function of the angle of inclination to find curves similar to their evolutes. We interpret the evolute of a plane curve as the caustic (envelope) of light rays normal to it, and study the Euler’s problem for general caustics. The resulting curves are characterized explicitly in two special cases: when the rays are at a constant angle to the curve (skew evolutes) and when they are reflections of parallel rays by it (caustics by reflection). Aside from the analogs of classical solutions known to Euler (logarithmic spirals, cycloids), we encounter infinite-dimensional families of self-similar curves that solve delay differential equations. (Received September 15, 2019)