Sherry Sarkar* (sherry.sarkar@outlook.com), Alex Xue and Pablo Soberón. Quantitative combinatorial geometry for concave functions.

Helly’s theorem is a fundamental result about the intersection properties of convex sets. It states that given $n$ convex sets in $\mathbb{R}^d$, if the intersection of every $d+1$ of the sets is non-empty, then the intersection of all the convex sets is non-empty.

In 1982, Báráný, Katchalski, and Pach proved a volumetric extension of Helly’s theorem stating that if the intersection of every $2d$ of the convex sets has volume at least one, then the volume of the intersection of all the sets is at least $d^{-2d^2}$.

In fact, one can show that a loss factor in the volume of the intersection is necessary. Our results characterize conditions that are sufficient for the intersection of a family of convex sets to contain a “witness set” which is large under some concave or log-concave measure. The possible witness sets include ellipsoids, zonotopes, and $H$-convex sets. Our results also bound the complexity of finding the best approximation of a family of convex sets by a single zonotope or by a single $H$-convex set. We obtain colorful and fractional variants of all our Helly-type theorems and also exact quantitative versions of Tverberg’s theorems. (Received September 17, 2019)