An acyclic orientation (AO) of an undirected graph is an assignment of direction to each of its edges without introducing a directed cycle. We study the enumeration of AOs for complete multipartite graphs. Our results include: an explicit formula for the number of AOs of complete multipartite graphs, which answers a question raised by Cameron et. al. (2014). We also provide a bijection between AOs of complete bipartite graphs with a fixed unique sink vertex and permutations with a prescribed exceedance set, relating two combinatorial objects not previously known to be connected.

Our techniques involve counting AOs on partially unlabeled complete multipartite graphs (i.e. counting AOs up to certain isomorphisms), then recovering the total count through a relabeling process.

Enumerating AOs is of interest for the connections they share with graph coloring. Indeed, the number of AOs is given by the chromatic polynomial evaluation $|\chi_G(-1)|$. This enumeration problem is also studied in computer science as a $\#P$-complete Tutte polynomial evaluation with unknown approximability. Finally, it appears in biology as the enumeration of branched polymers, and statistical physics as the Ursell function. (Received September 17, 2019)