On correlation properties of random graph homomorphisms. Preliminary report.

Given a bipartite graph $G$ and an integer $q \geq 2$, pick a proper $q$-coloring of $G$ uniformly at random. An old, apparently folkloric result says that the resultant distribution exhibits pairwise positive correlation, in the sense that if $x, y$ are two vertices in the same color class of $G$, knowing $x$ is red makes $y$ more likely to be red. We may naturally ask whether this correlation property also holds for random homomorphisms from $G$ to some more general class of target graphs $H$. It turns out that there are pairs $(G, H)$ for which it does not hold, leaving the question: what are the necessary restrictions on $G$ and/or $H$ to make it hold? We will discuss some possible answers to this question and state a few recent results. (Received August 09, 2004)