Permutations and ordinals: Wilf equivalence extended.

We reinterpret the idea of pattern-avoiding permutations or forbidden subsequences in terms of a subpattern relation and families closed under subpatterns. We thereby obtain a natural restatement of the classical question of Wilf equivalence which extends naturally to a broader question, and we answer the new, broader question in a variety of cases. We distinguish the family $S_n(132, 231)$ as having a strictly richer structure than the Wilf-equivalent family $S_n(132, 213)$.

To approach the new question, we introduce a new ordinal-valued measure of the size of a family, the depth, complementing the classical Wilf size. We prove a general theorem for computing depths, and apply it to several families including $S_n(132, 231)$, $S_n(132, 213)$, and $S_n(132)$. (Received October 05, 2004)