Hilbert’s wonderful eight-point construction. Preliminary report.

In 1888, Hilbert gave a construction of positive semidefinite ternary sextics with eight zeros in general position which cannot be written as a sum of squares of cubic forms. Unfortunately, the conditions on the zeros precluded any specific examples which could be easily calculated. It was not until 80 years later that R. M. Robinson relaxed the conditions in one special case, and gave an explicit form of this kind. We show that Hilbert’s construction always works under these more relaxed conditions, and give many specific examples. In particular, we derive a large class of extremal psd ternary sextics with 10 zeros. (Received October 04, 2004)