Meeting: 1003, Atlanta, Georgia, SS 35A, AMS-MAA Special Session on Tropical Geometry, I

1003-15-735 **Peter Butkovic*** (`p.butkovic@bham.ac.uk`), School of Mathematics, The University of Birmingham, Edgbaston, B15 2TT Birmingham, England. *On the strong regularity of matrices in max-algebra.*

Let $a \oplus b = \max(a, b)$ and $a \otimes b = a + b$ for $a, b \in \mathbb{R}$. Extend $(\oplus, \otimes)$ to matrices in the same way as in linear algebra.

Let $A = (a_{ij}) \in \mathbb{R}^{n \times n}$. Then A is called *strongly regular* if there is a $b \in \mathbb{R}^n$ such that the system $A \otimes x = b$ has exactly one solution. If P_n is the set of all permutations of the set $N = \{1, ..., n\}$ and $\pi \in P_n$ then the weight of π w.r.t. A is $w(A, \pi) = \sum_{i \in N} a_{i,\pi(i)}$. A is strongly regular if and only if there is exactly one permutation of maximal weight w.r.t. A. The set of all vectors b for which the system $A \otimes x = b$ has one exactly solution is called the *simple image set* $(S(A))$. The task of finding the simple image set can easily be transformed to the same question for normal matrices. If A is normal then $S(A)$ is the interior of the eigenspace of A.

Let $(G, \otimes, \leq)$ be a linearly ordered commutative group and define $a \oplus b = \max(a, b)$. The above criterion of strong regularity does not hold in general unless $(G, \leq)$ is dense but an $O(n^3)$ algorithm for checking the strong regularity in general groups exists. (Received October 02, 2004)