Let $H$ be a finite group with a normal, self-centralizing, elementary abelian $p$-subgroup $V$, and let $k(H)$ denote the number of conjugacy classes of $H$. Can one characterize all $H$ such that $k(H) > |V|$? The classical $k(GV)$-problem (solved very recently) addresses the case where $p$ is coprime to $|V|$, and the interest in it is motivated by its connection to the $k(B)$-conjecture of R. Brauer. We consider the general case where $p$ may divide $|V|$ – it is related to a recent conjecture of G. R. Robinson that bounds the numbers and heights of characters in $p$-constrained groups, and some other applications as well. Assuming $G := H/V$ is almost quasisimple and $O_p(G) = 1$, we show that either

(i) $k(H) < |V|/2$, or

(ii) $G$ belongs to an explicit list of “small” groups, and every composition factor $W$ of the $G$-module $V$ has $|W|$ bounded in terms of $G$, or

(iii) $G$ is a finite classical group in characteristic $p$, and every composition factor of the $G$-module $V$ is quasi-equivalent to the natural module of $G$. (Received September 21, 2004)