Let $D$ be the sub-Hopf algebra of the mod 2 Steenrod algebra, $A$, generated by the $P_t^*$'s with $s < t$. We filter $D$ by powers of the augmentation ideal and use the corresponding spectral sequence to calculate $H^1(D; \mathbb{F}_2)$ and $H^2(D; \mathbb{F}_2)$. The cohomology of $D$ is of interest because it can give rise to information about $H^*(A; \mathbb{F}_2)$ via a theorem by Palmieri which establishes an $F$-isomorphism between $H^*(D; \mathbb{F}_2)^{A/D}$ and $H^*(A; \mathbb{F}_2)$. The cohomology of the Steenrod algebra is in turn the $E_2$-term of the Adams spectral sequence converging to the stable homotopy groups of spheres. (Received October 03, 2004)