

1014-03-1564

**William C Calhoun\*** (wcalhoun@bloomu.edu), Bloomsburg University of Pennsylvania, Dept. of Math., Comp. Sci. and Stats., Bloomsburg, PA 17815. *Degrees of Monotone Complexity.*

Levin and Schnorr (independently) introduced the monotone complexity,  $K_m(\alpha)$ , of a binary string  $\alpha$ . We use monotone complexity to define the relative complexity (or relative randomness) of reals. We define a partial ordering  $\leq_{K_m}$  on  $2^\omega$  by  $\alpha \leq_{K_m} \beta$  iff there is a constant  $c$  such that  $K_m(\alpha \upharpoonright n) \leq K_m(\beta \upharpoonright n) + c$  for all  $n$ . The monotone degree of  $\alpha$  is the set of all  $\beta$  such that  $\alpha \leq_{K_m} \beta$  and  $\beta \leq_{K_m} \alpha$ . We show the monotone degrees contain an antichain of size  $2^{\aleph_0}$ , a countable dense linear ordering, and a minimal pair. We also show there is no minimal computably enumerable monotone degree.

Downey, Hirschfeldt, LaForte, Nies and others have studied a similar structure, the  $H$ -degrees, where  $H$  is the prefix-free Kolmogorov complexity. A minimal pair of  $H$ -degrees was constructed by Csima and Montalbán. Of particular interest are the noncomputable *trivial* reals, first constructed by Solovay. We define a real to be  $(K_m, H)$ -trivial if for some constant  $c$ ,  $K_m(\alpha \upharpoonright n) \leq H(1^n) + c$  for all  $n$ . We show every  $K_m$ -minimal real is  $(K_m, H)$ -trivial. (Received September 28, 2005)