1014-05-554 **Jack W Huizenga*** (huizenga@uchicago.edu). *On caps in* $(\mathbb{Z}/n\mathbb{Z})^2$.

A *line* in $(\mathbb{Z}/n\mathbb{Z})^2$ is any translate of a cyclic subgroup of order n. A *cap* is a subset $X \subset (\mathbb{Z}/n\mathbb{Z})^2$ which contains no collinear triple, and a cap is said to be *complete* if it is maximal with respect to set-theoretic inclusion. There are two natural extremal questions in the study of caps. First, what is the maximum size $\Psi(n)$ of a cap in $(\mathbb{Z}/n\mathbb{Z})^2$? On the other hand, what is the minimum size $\Phi(n)$ of a complete cap in $(\mathbb{Z}/n\mathbb{Z})^2$? These questions are closely related to analogous questions in the study of finite projective planes. We determine the following bounds on $\Phi(n)$ and $\Psi(n)$:

1. If p is the smallest prime divisor of n, then

$$\max\{4, \sqrt{2p} + \frac{1}{2}\} \leq \Phi(n) \leq \max\{4, p+1\}.$$

2. If q is a prime which divides n exactly a times, then

$$2 + \sum_{p|n}(p-1) \leq \Psi(n) \leq n \cdot (1 + q^{-\lceil (a+1)/2 \rceil} + q^{-a}),$$

   where the sum on the left is over all distinct prime divisors of n.

We also pose many interesting open questions for further study in this area. (Received September 20, 2005)