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Clustering of data sets must often be based on measurements taking place in ordered structures more general than the non-negative reals. This leads naturally to the study of residuated mappings from poset  $P$  to poset  $Q$ , and generalizations thereof. When the posets are complete lattices, we study the residuated approximation  $\rho_f$  of a map  $f: P \rightarrow Q$ ;  $\rho_f$  is the largest residuated mapping dominated by  $f$ . If  $P$  is a completely distributive lattice we give a formula showing how  $\rho_f$  may be calculated from  $f$ ; and, in the finite case, present an algorithm for computing  $\rho_f$ . This formula, the algorithm, and other results are based on the study of the mappings  $f^{(+)}$  and  $f^{(-)}$  defined (when possible) for  $q \in Q$  by  $f^{(+)}(q) = \bigvee\{p \in P : f(p) \leq q\}$ , and dually  $f^{(-)}(q) = \bigwedge\{p \in P : f(p) \geq q\}$ . If  $k \leq f$  is residuated, then  $k \leq f^{(+)(-)} = (f^{(+)})^{(-)} = \sigma_f$ . The map  $\sigma_f$  is called the *shadow* of  $f$ . It was introduced and studied in 1988 by H. Andréka, R. J. Greechie, and G. E Strecker. (Received September 18, 2005)