G.W. Anderson and the current authors published a new criterion for linear independence over function fields $k = \mathbb{F}_q(t)$ in Ann. Math. 160(2004), 237-313. This criterion provided a basis for establishing the function field analogue of Rohrlich’s conjecture on the algebraic relations on special Gamma values $\Gamma(a)$, but now for Thakur’s geometric Gamma function.

The criterion deals with certain (column) vectors $\psi(t)$ of entire functions satisfying functional equations involving the replacement of their power series coefficients by $q$th roots. The criterion asserts that if $\psi$ satisfies the criterion and if $\rho$ is a (row) vector with entries from $\bar{k}$ such that $\rho \psi(T) = 0$, then this is because there is a row vector $P(t)$ with entries from $\bar{k}[t]$ such that both

$$P(T) = \rho, \quad P(t) \psi(t) = 0.$$ 

In this talk, we present a general quantitative version of this criterion. In particular, when $\rho \psi(T) \neq 0$, we give an explicit lower bound on $|\rho \psi(T)|_\infty$, where $|\cdot|_\infty$ is the valuation such that $|T|_\infty = q$, in terms of the size of $\rho$, the maximum valuation of any conjugate of any entry of $\rho$. (Received September 27, 2005)